Journal of Communications Technology and Electronics, Vol. 43, No. 12, 1998, pp. 1388–1396. Translated from Radiotekhnika i Elektronika, Vol. 43, No. 12, 1998, pp. 1486–1494.

THEORY AND METHODS OF SIGNAL PROCESSING

One-Stage Estimation of the Location of a Radio Source by a Passive System

A. V. Dubrovin and Yu. G. Sosulin

Received January 21, 1994

Abstract—The problems of analysis and implementation of the algorithm of one-stage estimation of the radiator location are considered. The method of calculating the covariance error matrix for the one-stage algorithm is proposed. The comparative analysis of one- and two-stage algorithms is performed. It is shown that they have equal accuracy when the signal-to-noise ratio is above a certain threshold value. The advantage in the estimation accuracy of the one-stage algorithm in comparison with the two-stage algorithm is revealed at low values of the signal-to-noise ratio, when abnormal errors of measurements take place.

INTRODUCTION

Recently, wide-base passive systems of finding the location of radiators have been a subject of increasing interest. Here, we assume that in a wide-base system, the distance between the nearest receiving points is much greater than the wavelength of the accepted signal and commensurable with the range to the radiator.

The difference range-finding system may serve as an example of wide-base systems. In this system, the problem of finding the location of a radiator is usually solved in two stages. At the first stage, the time delays between the signals received by receivers located at different spatial points are estimated, and at the second stage, the coordinates of the radiator are estimated on the basis of the measured delays.

The united problem of estimating the location of a radiator is considered in [1]. However, Chernyak performed [2] the most complete study of this problem; he constructed the optimum (with respect to the maximum-likelihood criterion) one-stage algorithm of estimating the base delay vector, which is uniquely connected with the coordinates of the radiator. This method may be treated as an algorithm of estimation of the coordinates themselves. Unlike this paper, where the coordinates of the radiator are the measured parameters, in [2], the vector of independent delays is taken as a set of estimated parameters; the dimensionality of this vector equals two or three, respectively, when the situation on a plane or in space is considered.

In this paper, we consider the one-stage algorithm of optimum estimation of coordinates and the two-stage coordinate meter, in which, at the first stage, the delays are measured by the optimum maximum-likelihood method, and at the second stage, the coordinates are also estimated by an optimum method. We use a model of a signal that differs from the method proposed in [2]. In fact, we assume that the differences of phases of signals between the first and other receiving points are independent, rather than the differences of the unknown phases of signals at inputs of receiving points. We also consider a simpler heuristic technique of finding the solution that coincides essentially with the one-stage algorithm [2]; the difference is that, in our algorithm, the maximum is determined with the help of scanning in the space of coordinates and not in the space of base delays. We propose a procedure and calculate the potential accuracy for one- and two-stage algorithms.

The analysis has shown that one- and two-stage algorithms have identical accuracy when errors of measurements are small. However, it does not mean that the estimates of the coordinates of radiators obtained by the two-stage method are optimum. This is especially clear at small values of the small signal-to-noise ratio, when abnormal errors of measurements are revealed. The results of computer simulation show that the one-stage algorithm considered in this paper has a lower threshold signal-to-noise ratio that provides regular operation conditions of the meter.

1. STATEMENT OF THE PROBLEM

We have a radiator (R) that radiates a narrow-band signal $s(t)$ located in a frequency range covering the interval from f_1 to f_2. This signal is accepted by N spatially separated receivers Rec_n ($n = \overline{1, N}$) of the measuring system and is mixed with noise $\xi_n(t)$. The signal and noise are ergodic stationary independent Gaussian processes with zero mean. We assume that the received signals are either relayed or transmitted in a digital form to the central processing unit (CPU). Additional delays between the relayed signals caused by different distances between the receivers and CPU are compensated for, and the receiver timers are synchronized when analog-to-digital transformation of a signal is performed with an accuracy up to a certain nonrandom unknown quantity χ. The value of χ may be equal to several periods of the signal (one period equals $1/f_0$,

where $f_0 = (f_2 + f_1)/2$ is the center frequency), and the condition $\chi_{\max} 2\Delta_F \ll 1$ must be fulfilled, where $\Delta_F = (f_2 - f_1)/2$ and $2\Delta_F \ll f_0$. Thus, a model of signals observed in CPU can be presented in the form

$$y_n(t) = s_n(t) + \xi_n(t). \quad (1)$$

In the frequency domain, we have

$$\mathcal{Y}_n(f) = S_n(f) + \Xi_n(f),$$

where

$$\mathcal{Y}_n(f) = \int_{-\infty}^{+\infty} y_n(t)\exp(-j2\pi ft)dt = F(y_n(t));$$

$$S_n(f) = F(s_n(t)), \quad \Xi_n(f) = F(\xi_n(t));$$

F is the operator of the Fourier transform; $s_n(t) = a_n s(t - D_n)$ ($n = \overline{1, N}$) is the useful signal at the input of the nth receiver; a_n is the attenuation factor of signal $s(t)$ that specifies the attenuation level when this signal passes from radiator R to receiver Rec_n; $D_n = \tau_n(\mathbf{Z}) + \chi_n$ is the propagation time of signal $s(t)$ from R to Rec_n [$\tau_n(\mathbf{Z}) = R_n(\mathbf{Z})/c$)] plus the synchronization error (χ_n); $R_n(\mathbf{Z}) = \sqrt{(\mathbf{Z}_n - \mathbf{Z})^{\mathrm{T}}(\mathbf{Z}_n - \mathbf{Z})}$ is the distance from the radiator to the nth receiver; c is the propagation speed of the signal; $\mathbf{Z} = \|X_{\mathrm{R}}, Y_{\mathrm{R}}, Z_{\mathrm{R}}\|^{\mathrm{T}}$ are the coordinates of the radiator; $\mathbf{Z}_n = \|X_n, Y_n, Z_n\|^{\mathrm{T}}$ are the coordinates of the nth receiver; and T denotes the transposition.

We estimate the coordinates of the radiator by the maximum-likelihood method. Assume that the covariance functions of signal $s_n(t)$ and noise $\xi_n(t)$ ($n = \overline{1, N}$) are known:

$$K_{s_n}(\tau) = M[s_n(t)s_n(t-\tau)],$$
$$K_{\xi_n}(\tau) = M[\xi_n(t)\xi_n(t-\tau)], \quad M[s_n(t)] = 0, \quad (2)$$
$$M[\xi_n(t)] = 0.$$

According to model (1) of observed signals, the covariance functions of the useful signals proper differ only by the amplitude factors; therefore, all functions can be expressed through one function, for example, the first and then

$$K_{s_n}(\tau) = \alpha_n^2 K_{s_1}(\tau), \quad (3)$$

where $\alpha_n = a_n/a_1$, ($\alpha_1 = 1$).

The signal and noise are independent; therefore, the covariance function of the observed signal

$$K_{y_n y_n}(\tau) = M[y_n(t)y_n(t-\tau)] = K_{s_n}(\tau) + K_{\xi_n}(\tau). \quad (4)$$

In order to simplify subsequent analysis, we introduce the designation $K_s(\tau) \equiv K_{s_1}(\tau)$ and set

$$K_{s_{nk}}(\tau, D_{nk}) = M[s_n(t)s_k(t-\tau)] = \alpha_n\alpha_k K_s(\tau - D_{nk}), \quad (5)$$

where $D_{nk} = D_n - D_k = \tau_{nk}(\mathbf{Z}) + \chi_{nk}$; $\tau_{nk}(\mathbf{Z}) = \tau_{nl}(\mathbf{Z}) - \tau_{kl}(\mathbf{Z})$; $\tau_{nl}(\mathbf{Z}) = \tau_n(\mathbf{Z}) - \tau_l(\mathbf{Z})$; $\chi_{nk} = \chi_{nl} - \chi_{kl}$; $\chi_{nl} = \chi_n - \chi_l$; and χ_{nk} is the synchronization error of the nth receiver determined with respect to the kth receiver.

Taking into account (1) and (3) and the assumptions that the signal and noise are independent and noise factors are mutually independent, we obtain

$$K_{y_n y_k}(\tau, D_{nk}) = K_{s_{nk}}(\tau, D_{nk}). \quad (6)$$

Using (3), we derive the expressions for the spectral densities [3] of the considered signals and noise:

$$G_s(f) = \int_{-\infty}^{+\infty} K_s(\tau)\exp(-j2\pi f\tau)d\tau,$$
$$G_{\xi_n}(f) = \int_{-\infty}^{+\infty} K_{\xi_n}(\tau)\exp(-j2\pi f\tau)d\tau, \quad n = \overline{1, N}. \quad (7)$$

Taking into account (4)–(6), we obtain

$$G_{y_n y_n}(f) = \int_{-\infty}^{+\infty} K_{y_n y_n}(\tau)\exp(-j2\pi f\tau)d\tau = \alpha_n^2 G_s(f) + G_{\xi_n}(f),$$
$$G_{y_n y_k}(f, D_{nk}) = \int_{-\infty}^{+\infty} K_{y_n y_k}(\tau, D_{nk})\exp(-j2\pi f\tau)d\tau = \alpha_n\alpha_k G_s(f)\exp(-j2\pi f D_{nk}), \quad n \neq k. \quad (8)$$

Thus, the mutual spectral density $G_{y_n y_k}(f, D_{nk})$ and covariance function $K_{y_n y_k}(\tau, D_{nk})$ are known with an accuracy up to unknown delay D_{nk} that depends on the vector of the radiator coordinates $\mathbf{Z}$ and synchronization error χ_{nk}.

In order to obtain the representation for the likelihood function (LF), we assume that the signal is limited in time and occupies the interval from $-T_o/2$ to $+T_o/2$. Assume also that the observation time T_o is much greater than the maximum delay $D_{nk\max}$ ($n, k = \overline{1, N}$, $n \neq k$) and the width of the correlation functions (correlation intervals) of both the signal and noise. Then,

$$P(\mathbf{Y}|\mathbf{Z}, \mathbf{H}) = C_o \exp\left[-\frac{1}{2}\mathbf{Y}^{\mathrm{T}}\mathbf{G}^{-1}(\mathbf{Z}, \mathbf{H})\mathbf{Y}^*\right], \quad (9)$$

where $C_o = (2\pi)^{-IN/2}|\mathbf{G}(\mathbf{Z}, \mathbf{H})|^{-1/2}$ is the normalization

factor; $\mathbf{H} = \|\chi_{21}, \ldots, \chi_{N1}\|^{\mathrm{T}}$;

$$\mathbf{Y} = \left\|\mathbf{Y}^{\mathrm{T}}(0), \ldots, \mathbf{Y}^{\mathrm{T}}(I-1)\right\|^{\mathrm{T}};$$

$$\mathbf{Y}(i) = \left\|\mathcal{Y}_{\Delta 1}(i), \ldots, \mathcal{Y}_{\Delta N}(i)\right\|^{\mathrm{T}};$$

$$\mathcal{Y}_{\Delta n}(i) = \frac{1}{T_{\mathrm{o}}}\int_{-T_{\mathrm{o}}/2}^{+T_{\mathrm{o}}/2} y_n(t)\exp(-j2\pi i f_\Delta t)dt;$$

$$n = \overline{1, N}; \quad i = \overline{0, I-1}; \quad f_\Delta = \frac{1}{T_{\mathrm{o}}};$$

$\mathbf{G}(\mathbf{Z}, \mathbf{H}) = M(\mathbf{Y}^{\mathrm{T}}, \mathbf{Y}^*) = \mathrm{diag}(\mathbf{D}(\mathcal{G}_s(i)\mathbf{A}\mathbf{1}\mathbf{A} + \mathbf{G}_\xi(i))\mathbf{D}^*) = \mathrm{diag}(\mathcal{G}_s(i)\mathbf{D}\mathbf{A}(\mathbf{1} + \mathbf{C}(i))\mathbf{A}\mathbf{D}^*)$ is the $IN \times IN$ matrix consisting of $I \times I$ submatrices of dimensions $N \times N$ such that only the (i, j)th submatrices situated on the diagonal (at $i = j$, $i, j = \overline{0, I-1}$) are nonzero matrices; all elements of matrix $\mathbf{1}$ are equal to unity; $\mathbf{E}$ is the unit matrix; $\mathbf{A} = \mathrm{diag}(\alpha_n)$; $\mathbf{C}(i) = \mathrm{diag}\left(\frac{\mathcal{G}_{\xi_n}(i)}{\mathcal{G}_s(i)\alpha_n^2}\right)$; $\mathbf{G}_\xi(i) = \mathrm{diag}(\mathcal{G}_{\xi_n}(i))$; $\mathbf{D}(i) = \mathrm{diag}(\exp(-j2\pi i f_\Delta D_n))$ is the $N \times N$ matrix such that only its (n, k)th diagonal elements (at $n = k$, $n, k = \overline{1, N}$) are not equal to zero;

$$\mathcal{G}_{\xi_n}(i) = \frac{1}{T_{\mathrm{o}}}G_{\xi_n}(if_\Delta); \quad \mathcal{G}_s(i) = \frac{1}{T_{\mathrm{o}}}G_s(if_\Delta).$$

Consider the direct $N \times N$ matrix $\mathbf{Z} = \mathbf{1} + \mathbf{X}$, where $\mathbf{X}$ is the diagonal matrix with elements $X_1 \ldots X_N$. If we take into account that the inverse matrix $\mathbf{Z}^{-1}$ takes the form $\mathbf{Z}^{-1} = \mathbf{X}^{-1}(\mathbf{E} - \delta\mathbf{1}\mathbf{X}^{-1})$, $\delta = (1 + 1/X_1 + 1/X_2 + \ldots + 1/X_N)^{-1}$ (this expression can be easily derived by calculating the product of the direct inverse matrices), then we obtain

$$\mathbf{G}^{-1}(\mathbf{Z}, \mathbf{H}) = \mathrm{diag}\left(\mathcal{G}_s^{-1}(i)\mathbf{D}\mathbf{A}^{-1}\mathbf{C}^{-1}\left(\mathbf{E} - \left(1 + \sum_{l=1}^{N}\frac{\alpha_l^2\mathcal{G}_s(i)}{\mathcal{G}_{\xi_l}(i)}\right)^{-1}\mathbf{1}\mathbf{C}^{-1}\right)\mathbf{A}^{-1}\mathbf{D}^*\right) \tag{10}$$

$$= \mathrm{diag}\left(\mathbf{D}\left(\mathbf{G}_\xi^{-1}(i) - \left(1 + \sum_{l=1}^{N}\frac{\alpha_l^2\mathcal{G}_s(i)}{\mathcal{G}_{\xi_l}(i)}\right)^{-1}\mathbf{Q}(i)\right)\mathbf{D}^*\right),$$

where the elements of matrix $\mathbf{Q}(i)$ have the form $\mathcal{Q}_{nk}(i) = \frac{\alpha_n\alpha_k\mathcal{G}_s(i)}{\mathcal{G}_{\xi_n}(i)\mathcal{G}_{\xi_k}(i)}$ $(n, k = \overline{1, N})$.

Then,

$$P(\mathbf{Y}|\mathbf{Z}, \mathbf{H}) = C_{\mathrm{o}}\exp\left\{-\frac{1}{2}\left[\sum_{n=1}^{N}\sum_{i=0}^{I-1}\mathcal{Y}_{\Delta n}^*(i)\mathcal{Y}_{\Delta n}(i)\times\left(\mathcal{G}_{\xi_n}^{-1}(i) - \left(1 + \sum_{l=1}^{N}\frac{\alpha_l^2\mathcal{G}_s(i)}{\mathcal{G}_{\xi_l}(i)}\right)^{-1}\right)\mathcal{Q}_{nn}(i) - 2\sum_{n=1}^{N-1}\sum_{k=n+1}^{N}\sum_{i=0}^{I-1}\mathcal{Y}_{\Delta n}^*(i)Y_{\Delta k}(i)\times\left(1 + \sum_{l=1}^{N}\frac{\alpha_l^2\mathcal{G}_s(i)}{\mathcal{G}_{\xi_l}(i)}\right)^{-1}\mathcal{Q}_{nk}(i)\exp(j2\pi i f_\Delta D_{nk})\right]\right\}. \tag{11}$$

In relationship (11), normalization factor C_{o} and the first part of the expression under the exponent sign do not depend on $\mathbf{Z}$ and $\mathbf{H}$; therefore,

$$\ln(P(\mathbf{Y}|\mathbf{Z}, \mathbf{H})) = 2L(\mathbf{Z}, \mathbf{H}) + \mathrm{const},$$

where

$$L(\mathbf{Z}, \mathbf{H}) = \sum_{n=1}^{N-1}\sum_{k=n+1}^{N}\sum_{i=0}^{I-1}\mathcal{Y}_{\Delta n}^*(i)\mathcal{Y}_{\Delta k}(i)\times\left(1 + \sum_{l=1}^{N}\frac{\alpha_l^2\mathcal{G}_s(i)}{\mathcal{G}_{\xi_l}(i)}\right)^{-1}\mathcal{Q}_{nk}(i)\exp(j2\pi i f_\Delta D_{nk}). \tag{12}$$

If the observation time is sufficiently large, we have $\mathcal{Y}_{\Delta k}(i) \simeq \frac{1}{T_{\mathrm{o}}}\mathcal{Y}_k(if_\Delta)$.

Then,

$$L(\mathbf{Z}, \mathbf{H}) \simeq \sum_{n=1}^{N-1}\sum_{k=n+1}^{N}\sum_{i=0}^{I-1}\mathcal{Y}_n^*(if_\Delta)\mathcal{Y}_k(if_\Delta)\times\left(1 + \sum_{l=1}^{N}\frac{\alpha_l^2 G_s(if_\Delta)}{G_{\xi l}(if_\Delta)}\right)^{-1}\frac{\alpha_n\alpha_k G_s(if_\Delta)}{G_{\xi n}(if_\Delta)G_{\xi k}(if_\Delta)}\times\exp(j2\pi i f_\Delta D_{nk})f_\Delta.$$

Passing to the limit $f_\Delta \longrightarrow 0$, we obtain

$$L(\mathbf{Z}, \mathbf{H}) \simeq \sum_{n=1}^{N-1}\sum_{k=n+1}^{N}\int_{-\infty}^{+\infty}\mathcal{X}_n(f)\mathcal{X}_k^*(f)\times\exp\{j2\pi f D_{nk}\}df, \tag{13}$$

where $\mathcal{X}_n(f) = U_n(f)\mathcal{Y}_n(f)$; $U_n(f) = \frac{\alpha_n}{G_{\xi_n}(f)}\sqrt{\frac{G_s(f)}{1 + \sum_{i=1}^{N}q_n(f)}}$ is the frequency characteristic of the input filter in the

nth channel ($n = \overline{1, N}$, $\alpha_1 = 1$), and $q_n(f) = \alpha_n^2 \frac{G_s(f)}{G_{\xi_n}(f)}$ is the signal-to-noise ratio at the input of the nth receiver.

Taking into account the above assumption that $\chi_{\max} 2\Delta_F \ll 1$, we can write $2\pi f D_{nk} = \pi(f - f_0)\chi_{nk} + 2\pi f \tau_{nk}(\mathbf{Z}) + 2\pi f_0 \chi_{nk} \simeq 2\pi f \tau_{nk}(\mathbf{Z}) + 2\pi f_0 \chi_{nk}$ because, in this case, when f varies between f_1 and f_2, the influence of the term $2\pi(f - f_0)\chi_{nk} \ll \pi$ on the result of integration in negligible for all possible values of χ_{nk}. Thus, the synchronization error is revealed as a purely phase variation of the signal. Hence,

$$L(\mathbf{Z}, \mathbf{H}) = \sum_{n=1}^{N-1} \sum_{k=n+1}^{N} \cos(2\pi f_0 \chi_{nk} + \Phi_{nk}(\mathbf{Z}))|\mathfrak{L}_{nk}(\mathbf{Z})|, \quad (14)$$

where

$$\Phi_{nk}(\mathbf{Z}) = \arctan\{\mathrm{Im}(\mathfrak{L}_{nk}(\mathbf{Z}))/\mathrm{Re}(\mathfrak{L}_{nk}(\mathbf{Z}))\},$$

$$\mathrm{Re}(\mathfrak{L}_{nk}(\mathbf{Z})) = 2\int_0^{+\infty} |X_n(f)||X_k(f)| \times \cos(2\pi f \tau_{nk}(\mathbf{Z}) + \varphi_{nk}(f))df,$$

$$\mathrm{Im}(\mathfrak{L}_{nk}(\mathbf{Z})) = 2\int_0^{+\infty} |X_n(f)||X_k(f)| \times \sin(2\pi f \tau_{nk}(\mathbf{Z}) + \varphi_{nk}(f))df,$$

$$\varphi_{nk}(f) = \varphi_n(f) - \varphi_k(f);$$

$\varphi_i(f) = \arctan\{\mathrm{Im}(\mathscr{X}_i(f))/\mathrm{Re}(\mathscr{X}_i(f))\}$ is the phase-frequency characteristic of the ith signal ($i = n$ or k), and

$$|\mathfrak{L}_{nk}(\mathbf{Z})| = \sqrt{[\mathrm{Re}(\mathfrak{L}_{nk}(\mathbf{Z}))]^2 + [\mathrm{Im}(\mathfrak{L}_{nk}(\mathbf{Z}))]^2}$$

$$= 2\left\{\int_0^{+\infty}\int_0^{+\infty} |\mathscr{X}_n(f)||\mathscr{X}_k(f)||\mathscr{X}_n(\nu)||\mathscr{X}_k(\nu)| \times \cos(2\pi(f - \nu)\tau_{nk}(\mathbf{Z}) + (\varphi_{nk}(f) - \varphi_{nk}(\nu)))df d\nu\right\}^{1/2}.$$

Thus, we can represent the optimum estimation algorithm in the form

$$L(\mathbf{Z}, \mathbf{H}) = \max \text{ for } \mathbf{Z} = \hat{\mathbf{Z}}^{(0)}, \quad \mathbf{H} = \hat{\mathbf{H}}. \quad (15)$$

We see that in order to search for the maximum in (15), it is necessary to perform scanning both in the space $\mathfrak{H}$ ($\mathbf{H} \in \mathfrak{H}$) and $\mathfrak{Z}$ ($\mathbf{Z} \in \mathfrak{Z}$). According to the classical procedure, when the maximum-likelihood estimates of the vector $\mathbf{H}$ are obtained, it is necessary to substitute them into the likelihood function (14), and then to search for the maximum with respect to the coordinates of the radiator. A similar procedure is performed in [2] when the maximum with respect to delays is determined under the assumption that the differences of phases between all pairs of signals are independent (i.e., $\chi_{nk} \neq \chi_{n1} - \chi_{k1}$). In this paper, we assume that only the difference of phases between the signals received by the first and other receivers are independent (i.e., $\chi_{nk} \neq \chi_{n1} - \chi_{k1}$); as has been pointed out in [2], this assumption is more preferable from the practical viewpoint. Note that, for the model proposed in [2], the complete solution is obtained; however, for the case under consideration, it is rather difficult to obtain an explicit expression that does not contain a parasitic parameter $\mathbf{H}$. In this way, it is expedient to consider a simpler algorithm in which the search for the maximum is performed only in the space $\mathfrak{Z}$ using the procedure of maximization of the function

$$\mathscr{L}(\mathbf{Z}) = \sum_{n=1}^{N-1} \sum_{k=n+1}^{N} |\mathfrak{L}_{nk}(\mathbf{Z})|. \quad (16)$$

Thus, a heuristic (for the model of signal accepted in this paper) estimation algorithm is equivalent to the optimum (for the model of signal accepted in [2]) algorithm and has the form

$$\mathscr{L}(\mathbf{Z}) = \max \text{ for } \mathbf{Z} = \hat{\mathbf{Z}}^{(\mathrm{h})}. \quad (17)$$

The two-stage algorithm consists of two procedures: optimum measurement of the vector of delays and final optimum estimation of the radiator coordinates (the term optimum is understood in the sense that the sought after estimates are determined by the maximum-likelihood method). If we perform the same procedures that are used for deriving expressions (14) and (16) and carry out synthesis not with respect to $\mathbf{Z}$ but with respect to $\mathbf{T} = \|\tau_{21}, \ldots, \tau_{N1}\|^{\mathrm{T}}$, we obtain the following rules for estimating the vector of delays: the optimum algorithm

$$L_{\mathrm{T}}(\mathbf{T}, \mathbf{H}) = \sum_{n=1}^{N-1} \sum_{k=n+1}^{N} \cos(2\pi f_0 \chi_{nk} + \Phi_{nk}^{(\mathrm{T})}(\mathbf{T})) \times \left|\mathfrak{L}_{nk}^{(\mathrm{T})}(\mathbf{T})\right| = \max \quad (18)$$

for $\mathbf{T} = \hat{\mathbf{T}}^{(0)}$, $\mathbf{H} = \hat{\mathbf{H}}$

and the heuristic algorithm

$$\mathscr{L}_{\mathrm{T}}(\mathbf{T}) = \sum_{n=1}^{N-1} \sum_{k=n+1}^{N} \left|\mathfrak{L}_{nk}^{(\mathrm{T})}(\mathbf{T})\right| = \max \quad (19)$$

$$\text{for } \mathbf{T} = \hat{\mathbf{T}}^{(\mathrm{h})},$$

where

$$\Phi_{nk}^{(\mathrm{T})}(\mathbf{T}) = \arctan\{\mathrm{Im}(\mathfrak{L}_{nk}^{(\mathrm{T})}(\mathbf{T}))/\mathrm{Re}(\mathfrak{L}_{nk}^{(\mathrm{T})}(\mathbf{T}))\};$$

$$\mathrm{Re}(\mathfrak{L}_{nk}^{(\mathrm{T})}(\mathbf{T})) = 2\int_0^{+\infty} |\mathscr{K}_n(f)||\mathscr{K}_k(f)| \times \cos(2\pi f\tau_{nk} + \varphi_{nk}(f))df,$$

$$\mathrm{Im}(\mathfrak{L}_{nk}^{(\mathrm{T})}(\mathbf{T})) = 2\int_0^{+\infty} |\mathscr{K}_n(f)||\mathscr{K}_k(f)| \times \sin(2\pi f\tau_{nk} + \varphi_{nk}(f))df,$$

$$\tau_{nk} = \tau_{n1} - \tau_{k1},$$

$$\left|\mathfrak{L}_{nk}^{(\mathrm{T})}(\mathbf{T})\right| = \sqrt{[\mathrm{Re}(\mathfrak{L}_{nk}^{(\mathrm{T})}(\mathbf{T}))]^2 + [\mathrm{Im}(\mathfrak{L}_{nk}^{(\mathrm{T})}(\mathbf{T}))]^2},$$

and $\mathscr{K}_n(f)$ and $\varphi_{nk}(f)$ have the same sense as in (14).

The algorithm of the second stage—the algorithm of estimating the coordinates of the radiator—can be represented in the form [4]

$$\hat{\mathbf{Z}} = \mathbf{Z}_0 + (\mathbf{B}^{\mathrm{T}}\mathbf{S}^{-1}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{S}^{-1}(\hat{\mathbf{T}} - \mathbf{T}(\mathbf{Z}_0)), \qquad (20)$$

where $\mathbf{Z}_0 = \|X_0Y_0Z_0\|^{\mathrm{T}}$ are the coordinates of the supporting point; $\mathbf{T}(\mathbf{Z}_0) = \|\tau_{21}(\mathbf{Z}_0), \ldots, \tau_{N1}(\mathbf{Z}_0)\|^{\mathrm{T}}$ is the vector of delays corresponding to the supporting point [$\tau_{n1}(\mathbf{Z}_0) = \frac{1}{c}(R_n(\mathbf{Z}_0) - R_1(\mathbf{Z}_0))$; $R_n(\mathbf{Z}_0) = \sqrt{(\mathbf{Z}_n - \mathbf{Z}_0)^{\mathrm{T}}(\mathbf{Z}_n - \mathbf{Z}_0)}$; $\mathbf{Z}_n = \|X_nY_nZ_n\|^{\mathrm{T}}$ are the coordinate of the nth receiver]; $\mathbf{S} = M[(\hat{\mathbf{T}} - M(\hat{\mathbf{T}}))(\hat{\mathbf{T}} - M(\hat{\mathbf{T}}))^{\mathrm{T}}]$; $(\mathbf{B}^{\mathrm{T}}\mathbf{S}^{-1}\mathbf{B})^{-1} = M[(\hat{\mathbf{Z}} - M(\hat{\mathbf{Z}}))(\hat{\mathbf{Z}} - M(\hat{\mathbf{Z}}))^{\mathrm{T}}]$; and $B_{ni} = \frac{\partial\tau_{(n+1)1}(\mathbf{Z})}{\partial r_i}$ are the elements of matrix $\mathbf{B}$ [$n = \overline{1, N-1}$, $i = \overline{1, W}$, $r_1 = X_{\mathrm{R}}$, $r_2 = Y_{\mathrm{R}}$, $r_3 = Z_{\mathrm{R}}$, $W = 2$ for a plane, and $W = 3$ for a space].

We see that the differences between one-stage algorithm (15) [or (17)] and two-stage algorithm (18) [or (19)] and (20) are as follows: in the first case, the estimation of location of a radiator is performed by scanning with respect to coordinates $\mathbf{Z}$ in order to find an estimate $\hat{\mathbf{Z}}$ that maximizes LF; when the two-stage algorithm is applied, first, the vector of delays $\hat{\mathbf{T}}$ is estimated that corresponds to the maximum of its LF, and then the radiator coordinates are calculated by rule (20) using the obtained vector $\hat{\mathbf{T}}$.

2. ACCURACY OF ESTIMATING THE LOCATION OF THE RADIATOR

The potential accuracy of estimating the coordinates of a radiator by the optimum algorithm is determined by the Kramer–Rao matrix boundary $\mathbf{\Phi}_{\mathrm{KR}}$ equal to the inverse Fisher information matrix,

$$\mathbf{\Phi}_{\mathrm{KR}} = \begin{Vmatrix} \mathbf{\Phi}_{\mathrm{KR}}^{(1)} & \mathbf{\Phi}_{\mathrm{KR}}^{(2)} \\ (\mathbf{\Phi}_{\mathrm{KR}}^{(2)})^{\mathrm{T}} & \mathbf{\Phi}_{\mathrm{KR}}^{(3)} \end{Vmatrix} = \mathbf{\Phi}^{-1},$$

where $\mathbf{\Phi} = \begin{Vmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_2 \\ \mathbf{\Phi}_2^{\mathrm{T}} & \mathbf{\Phi}_3 \end{Vmatrix}$.

The elements of matrices $\mathbf{\Phi}_1$, $\mathbf{\Phi}_2$, and $\mathbf{\Phi}_3$ that form the Fisher information matrix $\mathbf{\Phi}$ are described as follows:

$$\mathscr{F}_{ij}^{(1)} = -M\left(\frac{\partial^2 L(\mathbf{Z}, \mathbf{H})}{\partial r_i \partial r_j}\right)_{\substack{\mathbf{Z}=\hat{\mathbf{Z}} \\ \mathbf{H}=\hat{\mathbf{H}}}}$$

$$= -M\left(\sum_{a=1}^{N-1}\sum_{b=1}^{N-1} \nu_{ab}^{(1)} \frac{\partial\tau_a}{\partial r_i}\frac{\partial\tau_b}{\partial r_j} + \sum_{a=1}^{N-1}\frac{\partial L(\mathbf{Z}, \mathbf{H})}{\partial\tau_a}\frac{\partial^2\tau_a}{\partial r_i \partial r_j}\right)_{\substack{\mathbf{Z}=\hat{\mathbf{Z}} \\ \mathbf{H}=\hat{\mathbf{H}}}},$$

$$\mathscr{F}_{ib}^{(2)} = -M\left(\frac{\partial^2 L(\mathbf{Z}, \mathbf{H})}{\partial r_i \partial\chi_b}\right)_{\substack{\mathbf{Z}=\hat{\mathbf{Z}} \\ \mathbf{H}=\hat{\mathbf{H}}}}$$

$$= -M\left(\sum_{a=1}^{N-1} \nu_{ab}^{(2)}\frac{\partial\tau_a}{\partial r_i}\right)_{\substack{\mathbf{Z}=\hat{\mathbf{Z}} \\ \mathbf{H}=\hat{\mathbf{H}}}},$$

$$\mathscr{F}_{ab}^{(3)} = -M(\nu_{ab}^{(3)})_{\substack{\mathbf{Z}=\hat{\mathbf{Z}} \\ \mathbf{H}=\hat{\mathbf{H}}}},$$

where

$$\nu_{ab}^{(1)} = \frac{\partial^2 L(\mathbf{Z}, \mathbf{H})}{\partial\tau_a \partial\tau_b}, \quad \nu_{ab}^{(2)} = \frac{\partial^2 L(\mathbf{Z}, \mathbf{H})}{\partial\tau_a \partial\chi_b},$$

$$\nu_{ab}^{(3)} = -M\left(\frac{\partial^2 L(\mathbf{Z}, \mathbf{H})}{\partial\chi_a \partial\chi_b}\right)_{\substack{\mathbf{Z}=\hat{\mathbf{Z}} \\ \mathbf{H}=\hat{\mathbf{H}}}}, \quad i, j = \overline{1, W},$$

$a, b = \overline{1, N-1}$, $\tau_a = \tau_{n1}(\mathbf{Z})$, $\chi_a = \chi_{n1}$, $n = a + 1$.

Differentiating (13) and calculating the expectation at the point $\hat{\mathbf{Z}}$, $\hat{\mathbf{H}}$, we obtain

$$M\left(\frac{\partial L(\mathbf{Z}, \mathbf{H})}{\partial\tau_a}\right)_{\substack{\mathbf{Z}=\hat{\mathbf{Z}} \\ \mathbf{H}=\hat{\mathbf{H}}}} = \sum_{\substack{i=1 \\ i\neq a+1}}^{N} \mathscr{V}_{i(a+1)}\mathrm{sgn}(a + 1 - i),$$

where

$$\mathscr{V}_{i(a+1)} = j\int_{-\infty}^{+\infty} \frac{f}{2\pi}\mathscr{P}_{i(a+1)}(f)df,$$

$$\mathrm{sgn}(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}; \quad a = \overline{1, N-1};$$

$$\mathcal{P}_{ab}(f) = T_o(2\pi)^2 q_a(f) q_b(f) \Big/ \left[1 + \sum_{n=1}^{N} q_n(f)\right],$$

$$\nu_{ab}^{(p)} = \begin{cases} -\sum\limits_{\substack{i=1 \\ i \neq b+1}}^{N} P_{i(b+1)}^{(p)}, & a = b, \\ P_{(a+1)(b+1)}^{(p)}, & a \neq b, \end{cases} \qquad a, b = \overline{1, N-1},$$

$$p = \overline{1, 3}.$$

Here,

$$P_{ab}^{(1)} = 2\int_0^{+\infty} f^2 P_{ab}(f)\,df, \quad P_{ab}^{(2)} = 2f_0 \int_0^{+\infty} f P_{ab}(f)\,df,$$

$$P_{ab}^{(3)} = 2f_0^2 \int_0^{+\infty} P_{ab}(f)\,df.$$

Taking into account the symmetry of function $q_i(f)$ with respect to the zero frequency, we obtain $\mathcal{V}_{ic} = 0$, which allows us to write the expression

$$\mathbf{\Phi} = \mathcal{B}^{\mathrm{T}} \mathbf{\Phi}_{\mathbf{T}} \mathcal{B}, \tag{21}$$

where

$$\mathbf{\Phi}_{\mathbf{T}} = \begin{Vmatrix} \mathbf{\Phi}_{\mathbf{T}}^{(1)} & \mathbf{\Phi}_{\mathbf{T}}^{(2)} \\ (\mathbf{\Phi}_{\mathbf{T}}^{(2)})^{\mathrm{T}} & \mathbf{\Phi}_{\mathbf{T}}^{(3)} \end{Vmatrix},$$

matrices $\mathbf{\Phi}_{\mathbf{T}}^{(1)}$, $\mathbf{\Phi}_{\mathbf{T}}^{(2)}$, and $\mathbf{\Phi}_{\mathbf{T}}^{(3)}$ consist of the elements $\nu_{ab}^{(1)}$, $\nu_{ab}^{(2)}$, and $\nu_{ab}^{(3)}$; $\mathcal{B} = \begin{Vmatrix} \mathbf{B} & \mathbf{O}_{N-1}^{N-1} \\ \mathbf{O}_W^{N-1} & \mathbf{E} \end{Vmatrix}$; $\mathbf{O}_{N-1}^{N-1}$ ($\mathbf{O}_W^{N-1}$) is the zero matrix consisting of $N-1$ rows and $N-1$ (W) columns; $\mathbf{E}$ is the unit $(N-1) \times (N-1)$ matrix; and the elements of matrix $\mathbf{B}$ are described in the explanation for (20).

In the matrix $\mathbf{\Phi}_{\mathrm{KR}}$, the element $\mathbf{\Phi}_{\mathrm{KR}}^{(1)}$ carries the information on the accuracy of determination of coordinates. Using the block Gaussian algorithm for inverting the matrix, we obtain

$$\mathbf{\Phi}_{\mathrm{KR}}^{(1)} = (\mathbf{B}^{\mathrm{T}} \mathbf{P}^{-1} \mathbf{B})^{-1}, \tag{22}$$

where $\mathbf{P} = (\mathbf{\Phi}_{\mathbf{T}}^{(1)} - \mathbf{\Phi}_{\mathbf{T}}^{(2)} (\mathbf{\Phi}_{\mathbf{T}}^{(3)})^{-1} (\mathbf{\Phi}_{\mathbf{T}}^{(2)})^{\mathrm{T}})^{-1}$ is the Kramer–Rao matrix boundary of the errors of estimating the vector of delays $\mathbf{T}$.

Thus, expression (22) enables one to calculate the accuracy parameters of the one-stage algorithm of estimating the coordinates. When implementing algorithm (20), the matrix boundary $\mathbf{P}$ can be used as matrix $\mathbf{S}$. In this case, the expression $(\mathbf{B}^{\mathrm{T}}\mathbf{S}^{-1}\mathbf{B})^{-1}$ that describes the covariance error matrix of estimating the coordinates by the two-stage method (when this method is applied, the vector of delays is estimated at the first stage, and the coordinates of the radiator are estimated at the second stage using the measured delays) coincides with (22). Hence, when errors of finding the location of a radiator are small, the potential accuracy of the two-stage and optimum one-stage methods (according to the latter method, the location is determined by the direct processing of signals received by sensors positioned at different spatial points) are equal, which confirms the results obtained in [2, 5, 6].

Let us analyze the properties of matrix $\mathbf{P}$. If the values of the signal-to-noise ratio are identical for all receivers, i.e., $q_i(f) = q_n(f) = q(f)$, $\mathcal{P}_{in}(f) = \mathcal{P}(f)$, and $P_{in}^{(k)} = P^{(k)}$ ($i \neq n$, $k = \overline{1, 3}$), then

$$\mathbf{P}^{-1} = P(N\mathbf{E} - \mathbf{1}). \tag{23}$$

Here, all elements of the $(N-1) \times (N-1)$ matrix $\mathbf{1}$ are equal to unity;

$$P = P^{(1)} - (P^{(2)})^2 / P^{(3)},$$

$$P^{(1)} = 2\int_0^{+\infty} f^2 \mathcal{P}(f)\,df, \quad P^{(2)} = 2f_0 \int_0^{+\infty} f \mathcal{P}(f)\,df,$$

$$P^{(3)} = 2f_0^2 \int_0^{+\infty} \mathcal{P}(f)\,df, \quad \mathcal{P}(f) = T_o(2\pi)^2 \frac{q^2(f)}{1 + Nq(f)}.$$

The Kramer–Rao matrix boundary of the errors of estimating the delays

$$\mathbf{P} = \frac{1}{PN}(\mathbf{E} + \mathbf{1}). \tag{24}$$

The diagonal elements of $\mathbf{P}$ are equal and can be represented in the form $\sigma_\tau^2 = 2/PN$. When the signal-to-noise ratio is uniform in the frequency band $2\Delta_F = F_2 - F_1$, $q(f) = q$, $\mathcal{P}(f) = \mathcal{P}$, $P^{(1)} = \left(\frac{4}{3}\Delta_F^3 + 4f_0^2\Delta_F\right)P$, $P^{(2)} = P^{(3)} = 4f_0^2\Delta_F\mathcal{P}$, and $P = \frac{4}{3}\Delta_F^3\mathcal{P}$, we can write

$$\sigma_\tau^2 = \frac{3(1 + Nq)}{8\pi^2 T_o \Delta_F^3 N q^2}. \tag{25}$$

Note that, for a sufficiently large value of the signal-to-noise ratio, the two- and one-stage algorithms have identical accuracy described by $\mathbf{\Phi}_{\mathrm{KR}}^{(1)}$. However, when the signal-to-noise ratio is small, $\mathbf{\Phi}_{\mathrm{KR}}^{(1)}$ will not describe the accuracy of determination of the radiator coordinates any longer. Since it is hardly possible to examine

such situations analytically, one should apply in this case the method of computer modeling.

We note also that there is no rigorous mathematical substantiation of heuristic algorithm (19) (for the model of signal accepted in this paper); therefore, the optimum and heuristic one-stage algorithms should be compared using the computer modeling.

3. RESULTS OF SIMULATION

The algorithms described in this paper were implemented using an IBM PC/AT. We compared algorithms (18) and (19) of measuring the vector of delays, as well as the two-stage (18) and (20) and one-stage (15) procedures of estimating the coordinates on a plane. The geometry of arrangement of radiator R and receivers Re is shown in Fig. 1. The dashed line denotes the domain, in which the location of the radiator is determined (it is known *a priori* that the radiator is located in the selected domain). The number of receivers N is equal to four. We assume that the signal and noise are Gaussian and mutually independent; their spectra are shown in Fig. 2; the observation time $T_o = 0.5$ ms; and the quantization interval $T_\Delta = 0.5/1024$ ms. The useful signal of the first receiver was generated in the frequency domain. The spectra of signals of other receivers were obtained by multiplying the first spectrum by the quantity $\exp(-j2\pi f_\Delta k D_{n1})$, where $f_\Delta = \dfrac{1}{T_o} = 2$ kHz ($k = \overline{0, 1023}$, $n = \overline{1, 4}$). Then, the useful signals were mixed with noises that were independently generated for each separate channel and come to the inputs of measuring units that implemented algorithms (18) and (19), (18) and (20), and (15). Here, it is necessary to tell about implementation of optimum algorithms (15) and (18). In spite of the fact that the scanning should also be performed in the space $\tilde{\mathfrak{H}}$ ($\mathbf{H} \in \tilde{\mathfrak{H}}$), it does not cause substantial temporal losses. In fact, $L(\mathbf{Z}, \mathbf{H})_{\mathbf{Z}=\text{const}}$ and $L(\mathbf{T}, \mathbf{H})_{\mathbf{T}=\text{const}}$ are smooth functions, and one can easily determine their maxima, when $\mathbf{H} = \hat{\mathbf{H}}$, by any numerical method performing several iterations. Thus, the search for maxima of functions $L(\mathbf{Z}, \hat{\mathbf{H}})$ and $L(\mathbf{T}, \hat{\mathbf{H}})$ is similar to the search for maxima of $L(\mathbf{Z})$ and $L(\mathbf{T})$.

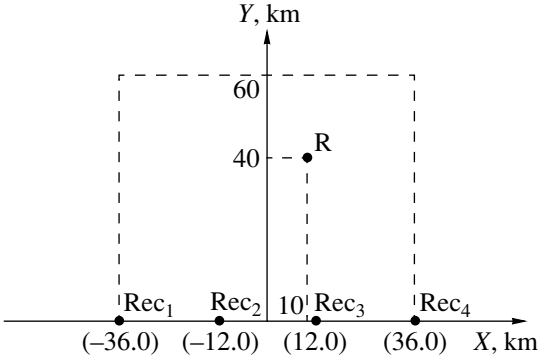

Fig. 1.

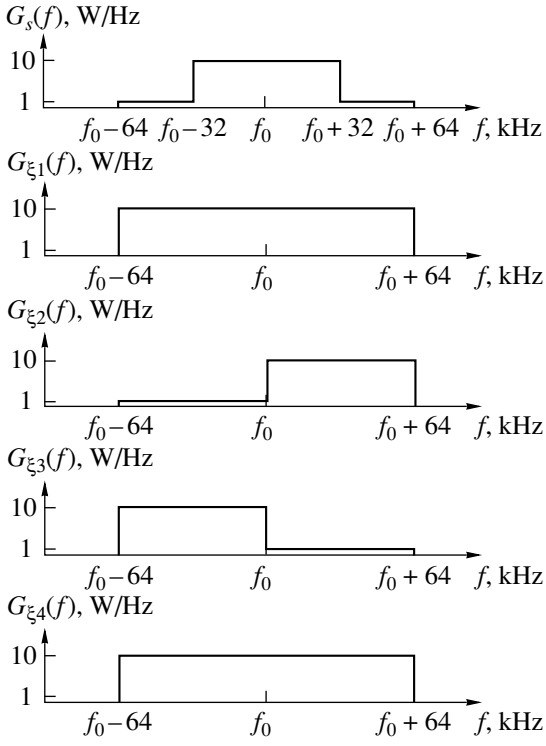

Fig. 2.

The table summarizes the results of comparative analysis of the optimum (18) and heuristic (19) algorithms of estimating the vector of delays $\mathbf{T} = \|\tau_{21}\tau_{31}\tau_{41}\|^{\mathrm{T}}$. Here, $\mu_{\log} = 10\log(\mu_p)$ and μ_p multiplies the spectral density of the signal power presented in Fig. 2 (the signal-to-noise ratio averaged with respect to power; unlike q, this quantity does not depend on f). The variance of measurements of the ith delay is estimated as $\mathrm{var}_i^{(h)} = \dfrac{1}{\mathcal{N}-1}\sum_{n=1}^{\mathcal{N}}(\hat{\tau}_{ni}^{(h)} - \tau_i^{\mathrm{t}})^2$, where $\mathcal{N}$ is the number of tests; τ_i^{t} is the true value of $\tau_{(i+1)1}$; $\hat{\tau}_{ni}^{(h)}$ is the estimate of $\tau_{(i+1)1}$ at the nth test by the optimum ($h = 1$) and heuristic ($h = 2$) methods ($i = \overline{1, 3}$). The theoretical values of variances, i.e., the diagonal elements of $\mathbf{P}$, are denoted by var_{Ti}. The number of tests $\mathcal{N} = 1000$. The boundaries of the 95% confidence interval are equal to $0.93\,\mathrm{var}_i^{(h)}$ and $1.08\,\mathrm{var}_i^{(h)}$ (see [7], p. 303).

As can be seen from the table, the data obtained by the method of modeling are sufficiently close to theoretical values of var_{Ti}. In addition, the optimum and heuristic algorithms have almost identical accuracy. Note that the same results were obtained when theoretical variances were investigated for a large array of other initial data. Thus, the results of simulation con-

Table

$\mu_{\log}$	$\mathrm{var}_1^{(1)}$, ms^2	$\mathrm{var}_2^{(1)}$, ms^2	$\mathrm{var}_3^{(1)}$, ms^2	$\mathrm{var}_1^{(2)}$, ms^2	$\mathrm{var}_2^{(2)}$, ms^2	$\mathrm{var}_3^{(2)}$, ms^2	var_{T1}, ms^2	var_{T2}, ms^2	var_{T3}, ms^2
20	9.98×10^{-9}	9.94×10^{-9}	1.40×10^{-8}	9.97×10^{-9}	9.95×10^{-9}	1.41×10^{-8}	9.96×10^{-9}	9.96×10^{-9}	1.37×10^{-8}
15	3.20×10^{-8}	3.22×10^{-8}	4.42×10^{-8}	3.21×10^{-8}	3.24×10^{-8}	4.41×10^{-8}	3.17×10^{-8}	3.17×10^{-8}	4.36×10^{-8}
10	1.05×10^{-7}	1.04×10^{-7}	1.41×10^{-7}	1.06×10^{-7}	1.07×10^{-7}	1.47×10^{-7}	1.03×10^{-7}	1.03×10^{-7}	1.41×10^{-7}
5	3.51×10^{-7}	3.50×10^{-7}	4.80×10^{-7}	3.52×10^{-7}	3.60×10^{-7}	4.81×10^{-7}	3.47×10^{-7}	3.47×10^{-7}	4.76×10^{-7}
0	1.57×10^{-6}	2.16×10^{-6}	7.40×10^{-4}	1.63×10^{-6}	2.26×10^{-6}	7.38×10^{-4}	1.28×10^{-6}	1.28×10^{-6}	1.75×10^{-6}

firm the conclusions obtained earlier by analytical methods.

The comparative analysis of the one-stage (15) and two-stage (18) and (20) algorithms was performed in the following manner. We assumed that, unlike the previous case, the signal and noise have rectangular spectra with the width $2\Delta_F$ = 128 kHz; the signal-to-noise ratio q is identical all four receivers. In the two-stage algorithm, first, the time delays between signals were estimated by optimum rule (18). Then, the coordinates of the radiator were estimated on the basis of the measured delays by known optimum algorithm (20). The one-stage algorithm is based on expression (15). Note that, for the minimum value $N = 3$, the one-stage algorithm has no advantages over the two-stage algorithm, since the second stage of the latter contains only one equation $\hat{\mathbf{Z}} = \mathbf{Z}_0$; thus, the two-stage algorithm consists actually of the first stage.

We have chosen the probability of abnormal estimate of the radiator coordinates as the performance criterion of these two algorithms (one- and two-stage); namely, the probability that the module of error $\sqrt{\Delta_x^2 + \Delta_y^2}$ ($\Delta_x = \hat{X} - X_\mathrm{R}$ and $\Delta_y = \hat{Y} - Y_\mathrm{R}$, where $\hat{X}$, $\hat{Y}$ and X_R, Y_R are, respectively, the measured and true coordinates of the radiator) exceeds the value $3\sqrt{\sigma_x^2 + \sigma_y^2}$, where σ_x^2 and σ_y^2 are the diagonal elements of matrix $\mathbf{\Phi}_{\mathrm{KR}}^{(1)}$.

Figure 3 shows the dependence of the probability of abnormal estimate P_abn on the signal-to-noise ratio q. We see that, for the chosen conditions, the synthesized one-stage algorithm (curve *1*) has an advantage over the two-stage algorithm (curve *2*) by approximately 2–3 dB. The given circumstance is explained by the fact that when the signal-to-noise ratio decreases below the threshold value, false modes begin to appear in addition to the main maximum of LF corresponding to true parameters of the signal; these modes are arbitrarily located in space of the search and their level is commensurable with the main extremum. In a broader aspect, this problem is discussed in [2].

In order to explain the reasons of this advantage, consider a typical situation when the level of noise has attained the value at which a false peak appears at a point $\hat{\mathbf{T}}$ that slightly exceeds the level of the main maximum corresponding to the point $\hat{\mathbf{T}}'$. This means malfunction of the system and obtaining arbitrary values of the measured parameters. In this case, we think that it is quite logical to select the maxima, at which the condition $\hat{\mathbf{Z}}_{jm} \approx \hat{\mathbf{Z}}_{ik}$ ($|i-j| + |k-m| \neq 0$, $i, j, n, m = \overline{2, N}$) is fulfilled, where $\hat{\mathbf{Z}}_{jm} = \|\hat{X}_{jm}\hat{Y}_{jm}\|^\mathrm{T}$ are the coordinates obtained at the points of intersection of the lines of position corresponding to the estimates of delays $\hat{\tau}_{j1}$ and $\hat{\tau}_{m1}$. This situation is illustrated by Fig. 4; the solid

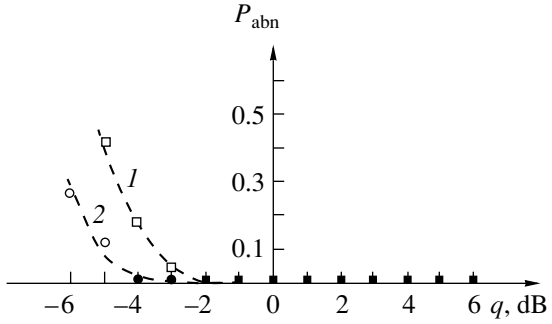

Fig. 3.

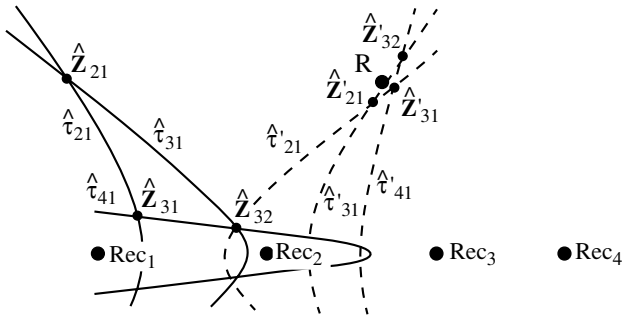

Fig. 4.

lines are hyperbolas that correspond to delays $\hat{\mathbf{T}} = \|\hat{\tau}_{21}\hat{\tau}_{31}\hat{\tau}_{41}\|^{\mathrm{T}}$, at which LF $L_{\mathbf{T}}(\mathbf{T}, \mathbf{H})$ [or $\mathscr{L}_{\mathbf{T}}(\mathbf{T})$] attains the global maximum.

We see that the coordinates $\hat{\mathbf{Z}}_{32}$, $\hat{\mathbf{Z}}_{42}$, and $\hat{\mathbf{Z}}_{43}$ calculated using these algorithms differ substantially. Hence, when estimating the coordinates at the second stage by algorithm (20), we determine the location of the radiator with a large (abnormal) error. The dashed hyperbolas are constructed for the second largest mode of function $L_{\mathbf{T}}(\mathbf{T}, \mathbf{H})$ [$\mathscr{L}_{\mathbf{T}}(\mathbf{T})$]. Here, the second mode of $L_{\mathbf{T}}(\mathbf{T}, \mathbf{H})$ ($\mathscr{L}_{\mathbf{T}}(\mathbf{T})$) attains its maximum at the point $\hat{\mathbf{T}}' = \|\hat{\tau}'_{21}\hat{\tau}'_{31}\hat{\tau}'_{41}\|^{\mathrm{T}}$. The points of intersection of the lines of position corresponding to the maximum of the second mode give the coordinates $\hat{\mathbf{Z}}'_{32}$, $\hat{\mathbf{Z}}'_{42}$, and $\hat{\mathbf{Z}}'_{43}$ that are concentrated around one point; i.e., $\hat{\mathbf{Z}}'_{32} \simeq \hat{\mathbf{Z}}'_{42} \simeq \hat{\mathbf{Z}}'_{43}$. This is an additional reason that confirms the following statement: in order to decide the coordinates of the radiator, it is expedient to choose the maximum of the second mode and not of the first in spite of the fact that the maximum of the first mode is greater than the maximum of the second mode (other techniques for eliminating abnormal errors are known; for references, see, for example, [2, Chapter 15.4]). The one-stage algorithm automatically uses this information. Therefore, the probability of normal estimation of the radiator coordinates obtained by the one-stage algorithm is higher in comparison with the two-stage algorithm.

CONCLUSION

The analysis of the optimum and heuristic algorithms performed in this paper shows that the measurement accuracy of both algorithms is almost equal.

The two-stage procedure of estimating the location of a radiator comprises estimation of the vector of delays between the accepted signals by the maximum-likelihood method and estimation (using the measured delays) of the coordinates of a radiator by the same method. This procedure may require an additional elimination of abnormal errors of measurements. In this case, it is expedient to analyze the operation quality of the algorithm of measuring the coordinates by comparing this algorithm with the one-stage procedure of estimating the location of the radiator; in fact, within the framework of the statement of the problem proposed in this paper, the latter procedure may be considered optimum on the whole. The analysis of the one-stage algorithm and its comparison with the two-stage procedure allows us to conclude that when the coordinates of low-power radiators are determined and the power of accepted signals is commensurable with noise, it is reasonable to use the one-stage algorithm that has a lower threshold signal-to-noise ratio providing regular operation conditions of the system. At the same time, when choosing an algorithm for finding the location of relatively powerful radiators, one should give preference to the algorithm which admits simpler implementation under specific operation conditions of a system of finding the location of radiators.

ACKNOWLEDGMENTS

We are grateful to V.S. Chernyak for helpful remarks.

REFERENCES

1. Kondrat'ev, V.S., Kotov, A.F., and Markov, L.N., *Mnogopozitsionnye radiotekhnicheskie sistemy* (Multiposition Radio Engineering Systems), Tsvetnov, V.V., Ed., Moscow: Radio i Svyaz', 1986.
2. Chernyak, V.S., *Mnogopozitsionnaya radiolokatsiya* (Multiposition Radio Location), Moscow: Radio i Svyaz', 1993.
3. Tikhonov, V.I., *Statisticheskaya radiotekhnika* (Statistical Radio Engineering), Moscow: Radio i Svyaz', 1982.
4. Torriery, D.J., *IEEE Trans. Aerosp. Electron Syst.*, 1984, vol. 20, no. 2, p. 183.
5. Hahn, W.R. and Tretter, S.A., *IEEE Trans. Inf. Technol.*, 1973, vol. 19, no. 5, p. 608.
6. Hahn, W.R., *J. Acoust. Soc. Am.*, 1975, vol. 58, no. 1, p. 201.
7. Abezgauz, G.G., Tron', A.P., Kopenkin, Yu.N., and Korovina, I.A., *Spravochnik po veroyatnostnym raschetam* (Handbook on Probabilistic Calculations), Moscow: Voenizdat, 1970.