ISSN 1064-2269, Journal of Communications Technology and Electronics, 2006, Vol. 51, No. 3, pp. 252–254. © Pleiades Publishing, Inc., 2006.
Original Russian Text © A.V. Dubrovin, 2006, published in Radiotekhnika i Elektronika, 2006, Vol. 51, No. 3, pp. 268–270.

ANTENNA AND FEEDER SYSTEMS

Potential Direction-Finding Accuracy of Systems with Antenna Arrays Configured as a Set of an Arbitrary Number of Rings

A. V. Dubrovin

Received March 16, 2005

Abstract—Configurations of antenna arrays for direction-finding systems in the form of a set of concentric circumferences are considered. An estimate of the potential accuracy of the measured bearing, which was obtained using the Cramer–Rao lower bound, is proposed. It is shown that in the case of antenna–feeder systems with antennas distributed uniformly over each circle, the potential accuracy is independent of the direction from which the received signal arrives.

PACS numbers: 84.40.Ba, 84.40.Ua

DOI: 10.1134/S1064226906030028

INTRODUCTION

Phase direction finders in which antenna arrays (AAs) are made up of antennas arranged in a circle are quite popular. Paper [1] describes a design that makes it possible to estimate the potential accuracy of a direction-finding system with an arbitrary configuration of AAs (see formula (25) in [1]). At the same time, there is a need for simpler expressions that allow one to very quickly and visually estimate the capabilities of various systems. Particular designs can be found, for example, in [2] (see formula (15.87)) for the case when the system contains two antennas, and in [1] (see formula (34)) for the case of four antennas.

This work proposes solutions following from formula (25) in [1] that visually demonstrate the potential of phase direction finders in application to circular AAs.

1. POTENTIAL ACCURACY OF MEASURED BEARING

In [1], an expression for the potential accuracy of measured angular coordinates of a radiator has been derived (see formula (25)). In practice, the situations are often quite important when it is necessary to estimate the potential of a direction-finding device under the condition when the angle of elevation of the received signal is zero. In order to obtain expressions convenient for analysis, it is necessary to make certain assumptions. We assume that both the signal and the noise are stationary, ergodic, and mutually independent Gaussian processes with zero mean. The signal is narrowband and falls in a frequency range from f_1 to f_2 with a $2\Delta_f$-spectrum width, and, in addition, $2\Delta_f \ll f_0$, where $\Delta_f = (f_2 - f_1)/2$ and $f_0 = (f_2 + f_1)/2$. The power spectral densities (PSDs) of the signal and the noise are rectangular functions; i.e., these densities are independent of frequency in a frequency band of $2\Delta_f$. In this case, the Cramer–Rao lower bound for errors in the measured angle of bearing looks as follows [1]:

$$\sigma_\alpha^2 = (\mathbf{B}_\alpha^T \mathbf{\Phi}_{T_\varphi}^{-1} \mathbf{B}_\alpha)^{-1}, \tag{1}$$

where $\mathbf{B}_\alpha^T = \|B_\alpha(1)\dots B_\alpha(M-1)\|$; $B_\alpha(\mu) = \frac{1}{c}[-(X_m - X_1)\sin(\alpha) + (Y_m - Y_1)\cos(\alpha)]$; $\mu = m - 1$; $m = \overline{2, M}$; X_i and Y_i are the coordinates of antenna A_i, $i = \overline{1, M}$; α is the direction toward the radiator; $\mathbf{\Phi}_{T_\varphi}^{-1} = P(M\mathbf{E} - \mathbf{1})$; $P = 16\pi^2 T_{\text{obs}} f_0^2 \Delta_f q^2/(1 + Mq)$; $\mathbf{E}$ is the unit matrix of order $(N-1) \times (N-1)$; $\mathbf{1}$ is the matrix of order $(N-1) \times (N-1)$ with all elements equal to 1; T_{obs} is the time of observation; $q = a^2 G_s/G_\xi$ is the signal-to-noise ratio; a is the amplitude of the received signal; G_s is the PSD of the signal; G_ξ is the PSD of the noise; M is the total number of antennas in the AA; and c is the propagation velocity of the signal.

Figure 1 shows the general view of a circular AA. It has the following characteristics: r_j is the radius of the jth ring; M_j is the number of antennas in the jth ring, where $= \overline{1, J}$; and J is the number of rings in the AA. We assume that the antennas are distributed uniformly along the circle and their coordinates can be represented as $X_i = r_j\cos(2\pi(k_j - 1)/M_j)$, $Y_i = r_j\sin(2\pi(k_j - 1)/M_j)$, where $k_j = \overline{1, M_j}$ and $i = k_j + \sum_{n=1}^{j-1} M_n$ ($\sum_{n=1}^{0} M_n \equiv 0$). If the central antenna is present, it is

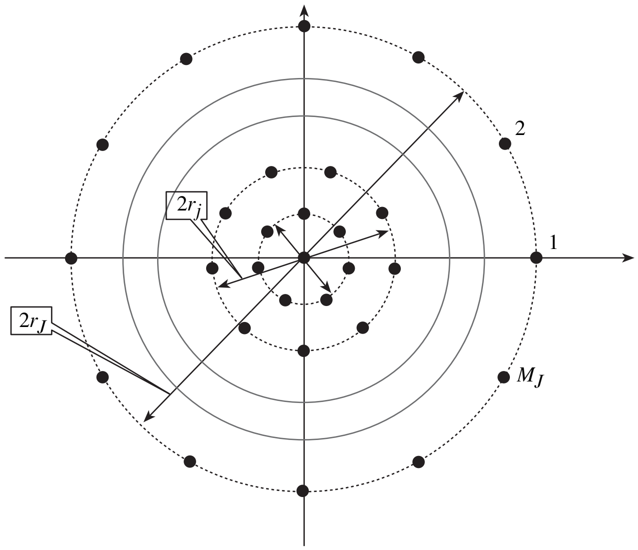

Fig. 1.

necessary to take into account that $r_1 = X_1 = Y_1 \equiv 0$ and $M_1 = 1$.

In view of the above, it is possible to write expression (1) in the convenient form

$$\sigma_\alpha = \frac{c\sqrt{1+Mq}}{\pi f_0 q\sqrt{8MM_r T_{\text{obs}}\Delta_f}}, \tag{2}$$

where $M = \sum_{j=1}^{J} M_j$ and $M_r = \sum_{j=1}^{J} M_j r_j^2$.

In order to attain a sufficiently visual representation of possibilities of a phase direction finder, we consider several configurations of the AAs used in practice (Figs. 2a–2c). For the selected configurations, we obtain from formula (2) the following respective relationships:

$$\sigma_\alpha = \frac{c\sqrt{1+Mq}}{\pi f_0 q r\sqrt{8M(M-1)T_{\text{obs}}\Delta_f}}, \tag{3a}$$

$$\sigma_\alpha = \frac{c\sqrt{1+(1+M_2+M_3)q}}{\pi f_0 q\sqrt{8(1+M_2+M_3)(M_2 r_2^2+M_3 r_3^2)T_{\text{obs}}\Delta_f}}$$
$$(M_1 = 1,\ r_1 = 0), \tag{3b}$$

$$\sigma_\alpha = \frac{c\sqrt{1+(M_1+M_2)q}}{\pi f_0 q\sqrt{8(M_1+M_2)(M_1 r_1^2+M_2 r_2^2)T_{\text{obs}}\Delta_f}}. \tag{3c}$$

2. RESULTS OF SIMULATION

In order to obtain an additional confirmation of the correctness of the mathematical manipulations and assumptions made in deriving the corresponding formulas, it is expedient to use computer simulation. For definiteness, we select the configuration of the antenna–feeder system (AFS) shown in Fig. 2b. In this case, the initial data are as follows: $r_1 = 0.12$ m, $r_2 = 0.6$ m, $f_0 = 50$ MHz, $2\Delta_f = 32$ kHz, $T_{\text{obs}} = 1$ ms, and $M_1 = M_2 = 3$. For an AFS of this kind, the calculation of the rms error is based on expression (3b). Figure 3 shows the dependence of σ_α on the signal-to-noise ratio $q_{\lg} = 10\log(q)$. The solid curve is calculated from formula (3b). Crosses denote values of sampled rms errors obtained as a result of simulation. The sampled rms error is calculated from the formula $\sigma_{\text{sml}\alpha} = \left[\frac{1}{N-1}\sum_{n=1}^{N}(\alpha_{\text{tr}} - \hat{\alpha}_n)\right]^{1/2}$, where α_{tr} is the true bearing value, $\hat{\alpha}_n$ is the bearing estimate in the nth test, and N is the number of tests (N = 100). As seen from the plot, for the selected conditions, the theoretical bound

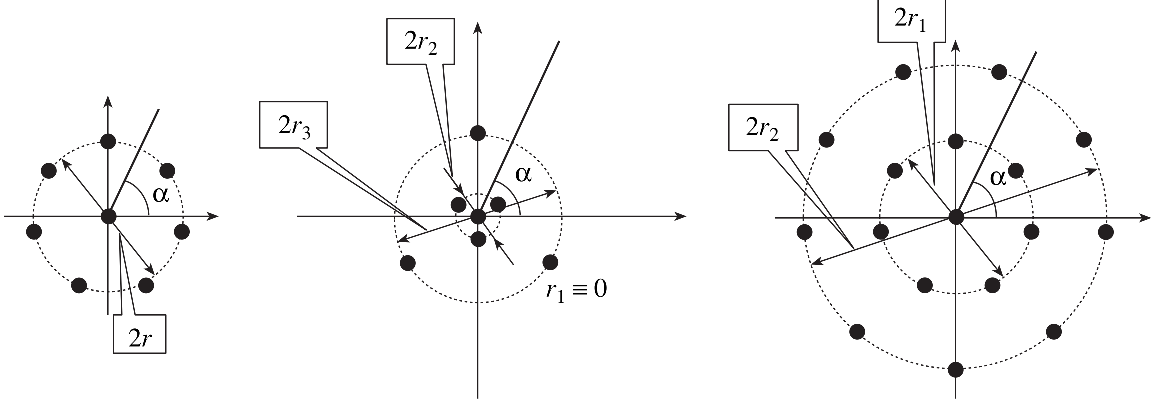

Fig. 2.

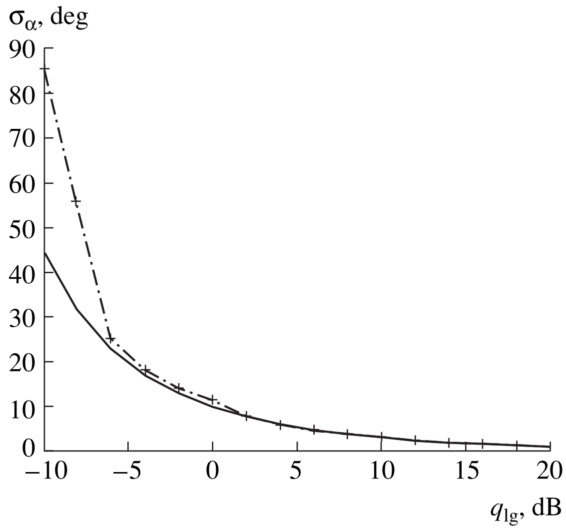

Fig. 3.

describes rather well the bearing measurement error up to a signal-to-ratio of –5 dB.

CONCLUSIONS

Solutions have been proposed that allow a rather visual estimation of the potential of direction-finding facilities containing circular antenna arrays. The simulation provides an additional confirmation of validity of the solutions.

REFERENCES

1. A. V. Dubrovin, and Yu. G. Sosulin, Radiotekhn. Elektron. **49** (2), 156 (2004) [J. Commun. Technol. Electron. **49**, 139 (2004)].
2. Ya. D. Shirman, and V. N. Manzhos, *Theory and Technique for Radar Data Processing in the Presence of Noise* (Radio i Svyaz', Moscow, 1981) [in Russian].